



IMONST 2 2021 JUNIOR CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 4 hours, from 10.00 am to 2.00 pm, 18 September 2021.
4. You need:
 - (a) An internet connection to download the question paper and submit your solutions.
 - (b) A4 papers (blank, not lined) for scratch papers and final scripts.
 - (c) Pens, pencils, erasers, geometric tools (compass, ruler).
 - (d) IMPORTANT: A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
6. Write your **full name, category, and school/institution** at the **top of the first page**.
7. Work on the solutions on your own. Any discussions, communication, or assistance from another person is not allowed and might result in disqualification.
8. Please upload your script in PDF format at <https://forms.gle/Ro2ZnXKaa346DhuA8> before 2.30pm.

IMONST 2 2021 (Junior)

Problem 1. Given a circle with center O . Points A and B lie on the circle such that triangle OBA is equilateral. Let C be a point outside the circle with $\angle ACB = 45^\circ$. Line CA intersects the circle at point D , and line CB intersects the circle at point E .

Find $\angle DBE$.

Diberi suatu bulatan dengan pusat O . Titik A dan B terletak pada bulatan tersebut sehinggakan segitiga OBA adalah segitiga sama sisi. Andaikan C suatu titik di luar bulatan tersebut dengan $\angle ACB = 45^\circ$. Garis CA bersilang dengan bulatan tersebut pada titik D , dan garis CB bersilang dengan bulatan tersebut pada titik E .

Cari $\angle DBE$.

Problem 2. Can we find positive integers a and b such that both $a^2 + b$ and $b^2 + a$ are perfect squares?

Wujudkah integer positif a dan b sehinggakan kedua-dua nombor $a^2 + b$ dan $b^2 + a$ adalah kuasa dua sempurna?

Problem 3. Given a sequence of positive integers

$$a_1, a_2, a_3, a_4, a_5, \dots$$

such that $a_2 > a_1$ and $a_{n+2} = 3a_{n+1} - 2a_n$ for all $n \geq 1$.

Prove that $a_{2021} > 2^{2019}$.

Diberi suatu jujukan integer positif

$$a_1, a_2, a_3, a_4, a_5, \dots$$

dengan $a_2 > a_1$ dan $a_{n+2} = 3a_{n+1} - 2a_n$ bagi semua $n \geq 1$.

Buktikan bahawa $a_{2021} > 2^{2019}$.

Problem 4. Find all values of n such that there exists a rectangle with integer side lengths, perimeter n , and area $2n$.

Cari semua nilai bagi n sehinggakan wujud suatu segiempat tepat dengan panjang sisi integer, perimeter n , dan luas $2n$.

Problem 5. Six teams participate in a hockey tournament. Each team plays once against every other team. In each game, a team is awarded 3 points for a win, 1 point for a draw, and 0 point for a loss.

After the tournament, the teams are ranked by total points. No two teams have the same total points. Each team (except the bottom team) has 2 points more than the team ranking one place lower.

Prove that the team that finished fourth has won two games and lost three games.

Enam pasukan bertanding dalam suatu kejohanan hoki. Setiap pasukan akan berlawan satu kali dengan setiap pasukan yang lain. Dalam setiap perlawanan, suatu pasukan akan mendapat 3 markah jika menang, 1 markah jika seri, dan 0 markah jika kalah.

Selepas kejohanan tersebut berakhir, semua pasukan disusun mengikut jumlah markah. Tidak ada dua pasukan yang mempunyai jumlah markah yang sama. Setiap pasukan (kecuali pasukan paling bawah sekali) mempunyai 2 markah lebih daripada pasukan yang berada satu tempat di bawahnya.

Buktikan bahawa pasukan yang mendapat tempat keempat telah menang sebanyak dua perlawanan dan kalah sebanyak tiga perlawanan.

Problem 6. Let x and y be two rational numbers such that

$$x^5 + y^5 = 2x^2y^2.$$

Prove that $\sqrt{1 - xy}$ is also a rational number.

Note: A rational number is a number that can be expressed as $\frac{a}{b}$, where a and b are integers ($b \neq 0$).

Andaikan x dan y adalah dua nombor nisbah sehinggakan

$$x^5 + y^5 = 2x^2y^2.$$

Buktikan bahawa $\sqrt{1 - xy}$ juga adalah suatu nombor nisbah.

Nota: Nombor nisbah ialah nombor yang boleh diungkapkan sebagai $\frac{a}{b}$, dengan a dan b integer ($b \neq 0$).

■ END OF PAPER ■