



IMONST 2 2021

PRIMARY CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 4 hours, from 10.00 am to 2.00 pm, 18 September 2021.
4. You need:
 - (a) An internet connection to download the question paper and submit your solutions.
 - (b) A4 papers (blank, not lined) for scratch papers and final scripts.
 - (c) Pens, pencils, erasers, geometric tools (compass, ruler).
 - (d) IMPORTANT: A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
6. Write your **full name, category, and school/institution** at the **top of the first page**.
7. Work on the solutions on your own. Any discussions, communication, or assistance from another person is not allowed and might result in disqualification.
8. Please upload your script in PDF format at <https://forms.gle/Ro2ZnXKaa346DhuA8> before 2.30pm.

IMONST 2 2021 (Primary)

Problem 1. An equilateral triangle ABC is divided by nine lines parallel to BC into ten bands that are equally wide. We colour the bands alternately red and blue, with the smallest band coloured red. The difference between the total area in red and the total area in blue is 20 cm^2 .

What is the area of triangle ABC ?

Suatu segitiga sama sisi ABC dibahagikan dengan sembilan garis yang selari dengan BC kepada sepuluh jalur dengan ketebalan yang sama. Kita mewarnakan jalur-jalur tersebut merah dan biru secara bergilir, dengan jalur yang paling kecil diwarnakan merah. Beza antara jumlah luas kawasan berwarna merah dan jumlah luas kawasan berwarna biru ialah 20 cm^2 .

Apakah luas bagi segitiga ABC ?

Problem 2. The five numbers a, b, c, d , and e satisfy the inequalities

$$a + b < c + d < e + a < b + c < d + e.$$

Among the five numbers, which one is the smallest, and which one is the largest?

Lima nombor a, b, c, d , dan e memenuhi ketaksamaan

$$a + b < c + d < e + a < b + c < d + e.$$

Antara lima nombor tersebut, nombor manakah yang paling kecil, dan nombor manakah yang paling besar?

Problem 3. Given a cube. On each edge of the cube, we write a number, either 1 or -1 . For each face of the cube, we multiply the four numbers on the edges of this face, and write the product on this face. Finally, we add all the eighteen numbers that we wrote down on the edges and faces of the cube.

What is the smallest possible sum that we can get?

Diberi sebuah kubus. Pada setiap sisi kubus tersebut, kita tuliskan suatu nombor, sama ada 1 atau -1 . Bagi setiap muka kubus tersebut, kita darabkan empat nombor pada sisi-sisi bagi muka tersebut, dan tuliskan hasil darabnya pada muka tersebut. Akhir sekali, kita tambahkan kesemua lapan belas nombor yang kita tuliskan pada semua sisi dan muka kubus tersebut.

Apakah hasil tambah paling kecil yang mungkin kita peroleh?

Problem 4. Given a circle with center O . Points A and B lie on the circle such that triangle OBA is equilateral. Let C be a point outside the circle with $\angle ACB = 45^\circ$. Line CA intersects the circle at point D , and line CB intersects the circle at point E .

Find $\angle DBE$.

Diberi suatu bulatan dengan pusat O . Titik A dan B terletak pada bulatan tersebut sehinggakan segitiga OBA adalah segitiga sama sisi. Andaikan C suatu titik di luar bulatan tersebut dengan $\angle ACB = 45^\circ$. Garis CA bersilang dengan bulatan tersebut pada titik D , dan garis CB bersilang dengan bulatan tersebut pada titik E .

Cari $\angle DBE$.

Problem 5. Can we find positive integers a and b such that both $a^2 + b$ and $b^2 + a$ are perfect squares?

Wujudkah integer positif a dan b sehinggakan kedua-dua nombor $a^2 + b$ dan $b^2 + a$ adalah kuasa dua sempurna?

Problem 6. Given a sequence of positive integers

$$a_1, a_2, a_3, a_4, a_5, \dots$$

such that $a_2 > a_1$ and $a_{n+2} = 3a_{n+1} - 2a_n$ for all $n \geq 1$.

Prove that $a_{2021} > 2^{2019}$.

Diberi suatu jujukan integer positif

$$a_1, a_2, a_3, a_4, a_5, \dots$$

dengan $a_2 > a_1$ dan $a_{n+2} = 3a_{n+1} - 2a_n$ bagi semua $n \geq 1$.

Buktikan bahawa $a_{2021} > 2^{2019}$.

■ END OF PAPER ■