



IMONST 2 2021

SENIOR CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 4 hours, from 10.00 am to 2.00 pm, 18 September 2021.
4. You need:
 - (a) An internet connection to download the question paper and submit your solutions.
 - (b) A4 papers (blank, not lined) for scratch papers and final scripts.
 - (c) Pens, pencils, erasers, geometric tools (compass, ruler).
 - (d) IMPORTANT: A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
6. Write your **full name, category, and school/institution** at the **top of the first page**.
7. Work on the solutions on your own. Any discussions, communication, or assistance from another person is not allowed and might result in disqualification.
8. Please upload your script in PDF format at <https://forms.gle/Ro2ZnXKaa346DhuA8> before 2.30pm.

IMONST 2 2021 (Senior)

Problem 1. Find all values of n such that there exists a rectangle with integer side lengths, perimeter n , and area $2n$.

Cari semua nilai bagi n sehinggakan wujud suatu segiempat tepat dengan panjang sisi integer, perimeter n dan luas $2n$.

Problem 2. Six teams participate in a hockey tournament. Each team plays once against every other team. In each game, a team is awarded 3 points for a win, 1 point for a draw, and 0 point for a loss.

After the tournament, the teams are ranked by total points. No two teams have the same total points. Each team (except the bottom team) has 2 points more than the team ranking one place lower.

Prove that the team that finished fourth has won two games and lost three games.

Enam pasukan bertanding dalam suatu kejohanan hoki. Setiap pasukan akan berlawan satu kali dengan setiap pasukan yang lain. Dalam setiap perlawanan, suatu pasukan akan mendapat 3 markah jika menang, 1 markah jika seri, dan 0 markah jika kalah.

Selepas kejohanan tersebut berakhir, semua pasukan disusun mengikut jumlah markah. Tidak ada dua pasukan yang mempunyai jumlah markah yang sama. Setiap pasukan (kecuali pasukan paling bawah sekali) mempunyai 2 markah lebih daripada pasukan yang berada satu tempat di bawahnya.

Buktikan bahawa pasukan yang mendapat tempat keempat telah menang sebanyak dua perlawanan dan kalah sebanyak tiga perlawanan.

Problem 3. Let x and y be two rational numbers such that

$$x^5 + y^5 = 2x^2y^2.$$

Prove that $\sqrt{1-xy}$ is also a rational number.

Note: A rational number is a number that can be expressed as $\frac{a}{b}$, where a and b are integers ($b \neq 0$).

Andaikan x dan y adalah dua nombor nisbah sehinggakan

$$x^5 + y^5 = 2x^2y^2.$$

Buktikan bahawa $\sqrt{1-xy}$ juga adalah suatu nombor nisbah.

Nota: Nombor nisbah ialah nombor yang boleh diungkapkan sebagai $\frac{a}{b}$, dengan a dan b integer ($b \neq 0$).

Problem 4. Given an octagon such that all its interior angles are equal, and all its sides have integer lengths.

Prove that any pair of opposite sides have equal lengths.

Diberi suatu oktagon sehinggakan kesemua sudut dalamannya sama, dan kesemua sisinya mempunyai panjang integer.

Buktikan bahawa setiap pasangan sisi yang bertentangan mempunyai panjang yang sama.

Problem 5. There are n guests at a gathering. Any two guests are either friends or not friends. Every guest is friends with exactly four of the other guests. Whenever a guest is not friends with two other guests, those two other guests cannot be friends with each other either.

Determine all possible values of n .

Terdapat n orang tetamu pada suatu majlis. Setiap dua orang tetamu adalah sama ada berkawan atau tidak berkawan. Setiap tetamu berkawan dengan tepat empat orang tetamu lain. Bilamana seorang tetamu tidak berkawan dengan dua orang tetamu lain, kedua-dua tetamu tersebut juga tidak berkawan sesama mereka.

Tentukan semua nilai yang mungkin bagi n .

Problem 6. Prove that there is a positive integer m such that the number $5^{2021}m$ has no even digits (in its decimal representation).

Buktikan bahawa terdapat suatu integer positif m sehinggakan nombor $5^{2021}m$ tidak mempunyai digit genap (dalam perwakilan asas 10 bagi nombor tersebut).

■ END OF PAPER ■

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