



IMONST 2 2022 (SENIOR)

READ THESE INSTRUCTIONS CAREFULLY:

1. **Time: 4 hours** (10.00 am to 2.00 pm, Saturday 30 July 2022).
2. The paper contains **six (6) problems**. Make sure you have the paper for the correct category. Each problem is worth 7 points.
3. You need:
 - (a) Internet access to download the question and submit your solution (not allowed during the contest period).
 - (b) A4 papers, pen, pencil, eraser, compass, ruler.
 - (c) A scanner or phone scanner app (e.g., Adobe Scan) to scan your solution.

Not allowed: Calculators, electronic/internet devices, books, printed materials.

4. Write the solution **by hand**. Do not use a computer.
5. Write your **full name, category, and school name** on **top of the first page**.
6. Work on your own. Any evidence of cheating will result in disqualification and ban from future IMONST events.
7. After the contest, scan your solution and save it as **one PDF file**, with filename:
(**Category**)\space(**Full name**) Example: (Primary) Muhammad Zafree Zaharidan.
8. Upload your solution at this link **before 2.30pm**:

<https://tinyurl.com/imonst2sub>

Late submission will be rejected.

GOOD LUCK

IMONST 2 2022 (Senior)

Problem 1. Given a circle Γ with center I , and a triangle ABC with all its sides tangent to Γ . A line ℓ is drawn such that it bisects both the area and the perimeter of triangle ABC .

Prove that line ℓ passes through I .

Diberi suatu bulatan Γ dengan pusat I , dan suatu segi tiga ABC dengan semua sisinya tangen kepada Γ . Suatu garis ℓ dilukis supaya ia membahagi dua sama kedua-dua luas dan perimeter bagi segitiga ABC .

Buktikan bahawa garis ℓ melalui titik I .

Problem 2. It is known that there are n integers $a_1, a_2, \dots, a_n$ such that

$$a_1 + a_2 + \dots + a_n = 0 \quad \text{and} \quad a_1 \times a_2 \times \dots \times a_n = n.$$

Determine all possible values of n .

Diketahui bahawa wujud n integer $a_1, a_2, \dots, a_n$ sehinggakan

$$a_1 + a_2 + \dots + a_n = 0 \quad \text{dan} \quad a_1 \times a_2 \times \dots \times a_n = n.$$

Tentukan semua nilai yang mungkin bagi n .

Problem 3. Prove that there is a multiple of 2^{2022} that has 2022 digits, and can be written using digits 1 and 2 only.

Buktikan bahawa wujud suatu gandaan bagi 2^{2022} yang mempunyai 2022 digit, dan boleh ditulis dengan hanya menggunakan digit 1 dan 2 sahaja.

Problem 4. Given a pentagon $ABCDE$ with all its interior angles less than 180° .

Prove that if $\angle ABC = \angle ADE$ and $\angle ADB = \angle AEC$, then $\angle BAC = \angle DAE$.

Diberi suatu pentagon $ABCDE$ dengan kesemua sisi dalamnya kurang daripada 180° .

Buktikan bahawa jika $\angle ABC = \angle ADE$ dan $\angle ADB = \angle AEC$, maka $\angle BAC = \angle DAE$.

Problem 5. Let a , b , r , and s be positive integers ($a \geq 2$), where a and b have no common prime factor.

Prove that if $a^r + b^r$ is divisible by $a^s + b^s$, then r is divisible by s .

Andaikan a , b , r , dan s integer positif ($a \geq 2$), dengan a dan b tiada faktor perdana sepunya.

Buktikan bahawa jika $a^r + b^r$ boleh bahagi dengan $a^s + b^s$, maka r boleh bahagi dengan s .

Problem 6. A football league has n teams. Each team plays one game with every other team. Each win is awarded 2 points, each tie 1 point, and each loss 0 point.

After the league is over, the following statement is true: for every subset S of teams in the league, there is a team (which may or may not be in S) such that the total points the team obtained by playing all the teams in S is odd.

Prove that n is even.

Suatu liga bolasepak mempunyai n pasukan. Setiap pasukan bermain satu perlawanan dengan setiap pasukan yang lain. Setiap kemenangan diberikan 2 mata, setiap seri 1 mata, dan setiap kekalahan 0 mata.

Selepas liga tersebut tamat, pernyataan berikut adalah benar: bagi setiap subset S bagi pasukan dalam liga tersebut, terdapat satu pasukan (yang mungkin berada atau tidak berada dalam S) sehinggakan jumlah mata yang diperolehi pasukan tersebut melalui perlawanan dengan semua pasukan dalam S adalah ganjil.

Buktikan bahawa n adalah genap.

■ END OF PAPER ■