



IMONST 2 2023

SENIOR CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for the Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 4 hours, from 10.00 am to 2.00 pm, 21 October 2023.
4. You need:
 - (a) An internet connection to download the question paper and submit your solutions.
 - (b) To turn on the camera for the whole duration of the test.
 - (c) A4 papers (blank, not lined) for scratch papers and final scripts.
 - (d) Pens, pencils, erasers, geometric tools (compass, ruler).
 - (e) IMPORTANT: A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
6. Write your **full name, category, and school/institution** at the **top of the first page**.
7. Work on the solutions on your own. Any discussions, communication, or assistance from another person is not allowed and might result in disqualification.
8. Calculator is NOT allowed. This is NOT an open-book test.
9. Please upload your script in PDF format at <https://forms.gle/xr5ckkLyPyeoY5Vp6> before 2.30pm. Also, name the file as: Primary/Junior/Senior Your Name.
E.g. Senior - Suhaimi Ramly

IMONST 2 2023 (Senior)

Problem 1. Prove that there exist infinitely many positive integers with the following properties:

- it can be written as the sum of 2001 distinct perfect squares,
- it can be written as the sum of 2023 distinct positive perfect cubes.

Buktikan bahawa wujud tak terhingga banyaknya integer positif dengan sifat-sifat berikut:

- *ia boleh ditulis sebagai hasil tambah 2001 kuasa dua sempurna yang berbeza,*
- *ia boleh ditulis sebagai hasil tambah 2023 kuasa tiga sempurna positif yang berbeza.*

Problem 2. Prove that if x , y and n are positive integers such that

$$x^{2024} + y^{2024} = 2^n,$$

then $x = y$.

Buktikan bahawa jika x , y dan n adalah integer-integer positif sehinggakan

$$x^{2024} + y^{2024} = 2^n,$$

maka $x = y$.

Problem 3. Fix an integer $n \geq 1$. On a $m \times n$ chessboard, what is the minimum value of m such that n queens can be placed on the chessboard without any two attacking each other? (A queen can attack vertically, horizontally, or diagonally across any distance.)

Tetapkan suatu integer $n \geq 1$. Pada sebuah papan catur bersaiz $m \times n$, apakah nilai minimum bagi m supaya n permaisuri boleh diletakkan pada papan catur tersebut tanpa sebarang dua permaisuri menyerang sesama mereka? (Suatu permaisuri boleh menyerang secara menegak, melintang, dan menyerong pada sebarang jarak.)

Problem 4. Given a right angled triangle ABC with $\angle BAC = 90^\circ$. The points D, E, F lie on sides BC, CA, AB respectively so that AD is perpendicular to BC and EF is parallel to BC . A point G lies on side AC such that $AG = CE$.

Prove that $\angle GDF = 90^\circ$.

Diberi suatu segi tiga bersudut tegak ABC dengan $\angle BAC = 90^\circ$. Titik-titik D, E, F masing-masing berada pada sisi BC, CA, AB sehinggakan AD adalah serenjang dengan BC dan EF adalah selari dengan BC . Suatu titik G berada pada sisi AC sehinggakan $AG = CE$.

Buktikan bahawa $\angle GDF = 90^\circ$.

Problem 5. Find the smallest positive m such that if a, b, c are three side lengths of a triangle with $a^2 + b^2 > mc^2$, then c must be the length of shortest side.

Cari nombor positif terkecil m sehinggakan jika a, b, c adalah tiga panjang sisi bagi segi tiga dengan $a^2 + b^2 > mc^2$, maka c mestilah panjang sisi yang terpendek.

Problem 6. Ivan has a parallelogram whose interior angles are $60^\circ, 120^\circ, 60^\circ, 120^\circ$ respectively, and all side lengths are integers. Is it possible that one of the diagonals has length $\sqrt{2024}$?

Ivan mempunyai suatu sisi empat selari dengan sudut-sudut pedalaman masing-masing adalah $60^\circ, 120^\circ, 60^\circ, 120^\circ$, dan kesemua panjang sisi adalah integer. Adakah mungkin yang salah satu pepenjuru mempunyai panjang $\sqrt{2024}$?

■ END OF PAPER ■