IMONST 2 2023
PRIMARY CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for the Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 4 hours, from 10.00 am to 2.00 pm, 21 October 2023.
4. You need:
   (a) An internet connection to download the question paper and submit your solutions.
   (b) To turn on the camera for the whole duration of the test.
   (c) A4 papers (blank, not lined) for scratch papers and final scripts.
   (d) Pens, pencils, erasers, geometric tools (compass, ruler).
   (e) IMPORTANT: A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
6. Write your **full name, category, and school/institution** at the **top of the first page**.
7. Work on the solutions on your own. Any discussions, communication, or assistance from another person is not allowed and might result in disqualification.
8. Calculator is NOT allowed. This is NOT an open-book test.
9. Please upload your script in PDF format at `https://forms.gle/xr5ckkLyPyeoY5Vp6` before 2.30pm. Also, name the file as: Primary/Junior/Senior Your Name.
   E.g. Primary - Suhaimi Ramly

IMONST 2 2023 (Primary)

Problem 1. Prove that for all positive integers n, $1^3 + 2^3 + 3^3 + \cdots + n^3$ is a perfect square.

Buktikan untuk semua integer positif n bahawa $1^3 + 2^3 + 3^3 + \cdots + n^3$ ialah kuasa dua sempurna.

Problem 2. Ivan bought 50 cats consisting of five different breeds. He records the number of cats of each breed and after multiplying these five numbers he obtains the number 100000. How many cats of each breed does he have?

Ivan membeli 50 kucing yang terdiri daripada lima baka yang berbeza. Dia merekodkan bilangan kucing bagi setiap baka dan selepas mendarabkan lima nombor tersebut dia memperoleh nombor 100000. Berapakah bilangan kucing bagi setiap baka yang dia ada?

Problem 3. Find all pairs of positive integers (x, y), so that the number $x^3 + y^3$ is a prime.

Cari kesemua pasangan integer positif (x, y), supaya nombor $x^3 + y^3$ ialah perdana.

Problem 4. Given a right-angled triangle with hypotenuse 2024, find the maximal area of the triangle.

Diberi suatu segi tiga bersudut tegak dengan hipotenus 2024, cari luas maksimum bagi segi tiga tersebut.

Problem 5. Ruby writes the numbers $1, 2, 3, \ldots, 10$ on the whiteboard. In each move, she selects two distinct numbers, a and b, erases them, and replaces them with $a + b - 1$. She repeats this process until only one number, x, remains. What are all the possible values of x?

Ruby menulis nombor $1, 2, 3, \ldots, 10$ *pada papan putih. Pada setiap langkah, dia memilih dua nombor yang berbeza,* a *dan* b*, padamkan kedua-dua nombor tersebut, dan gantikan mereka dengan* $a + b - 1$*. Dia mengulangi proses ini sehingga hanya satu nombor,* x*, yang tinggal. Apakah kesemua nilai yang mungkin bagi* x*?*

Problem 6. Suppose $ABCD$ is a cyclic quadrilateral with $\angle ABC = \angle ADC = 90^\circ$. Let E and F be the feet of perpendiculars from A and C to BD respectively. Prove that $BE = DF$.

Andai $ABCD$ *suatu sisi empat kitaran dengan* $\angle ABC = \angle ADC = 90^\circ$*. Jadikan* E *dan* F *kaki bagi garis-garis serenjang masing-masing dari* A *dan* C *ke* BD*. Buktikan bahawa* $BE = DF$*.*

■ END OF PAPER ■