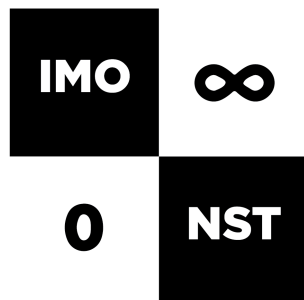




IMO National Selection Test
Ujian Pemilihan Kebangsaan IMO

IMO NATIONAL SELECTION TEST 2 2025

JUNIOR CATEGORY

8 NOVEMBER 2025

INSTRUCTIONS:

1. This paper consists of 6 essay questions, each worth 7 points. Answer **ALL** questions.
 2. Each question comes in English Language followed by *Bahasa Melayu*.
 3. The duration of this paper is 4 hours.
 4. Write your answers in the space provided. Show your working to obtain points. You are not required to rewrite the questions.
 5. Only basic utensils and geometric sets are allowed during the exam. Calculators, electronic/internet devices, books, and printed materials are **NOT ALLOWED**.
 6. Discussion, cheating, and guidance are **NOT ALLOWED** during the exam. Any evidence will result in disqualification.
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ENGLISH

- 1** Find all arithmetic progressions of positive integers $a_1, a_2, \dots$ such that for all n ,

$$\gcd(a_n, a_{2n}) = n$$

Note: $\gcd(x, y)$ denotes the greatest common divisor of x and y .

- 2** Let ABC be an acute triangle with $AB < AC$. Let D be any point on the perpendicular bisector of BC . The circumcircle of BCD intersects AB at E and AC at F . The line DE and DF intersects the line parallel to BC through A at G and H .

Prove that $GHFE$ is cyclic quadrilateral.

- 3** There 100 boxes. In each box, Ivan places a red ball into it, then for all $1 \leq i \leq 100$, he places i blue balls into the i -th box. Now, from each box, he picks one ball at random.

What is the probability that he will get odd number of red balls?

- 4** Given an n by n grid, some of the squares are placed with a garlic. A Dracula wants to move from the bottom left square to the top right square of the grid. In each step Dracula can only move to a square directly adjacent to it, and it cannot be directly adjacent or be on a square that contains garlic.

Given that Dracula succeeded in doing so, what is the maximum number of squares that can contain a garlic?

- 5** Let $ABCD$ be a cyclic quadrilateral with its diagonals AC and BD intersecting at P . Points W, X, Y, Z are constructed outside of quadrilateral $ABCD$, such that triangles WAB, XBC, YCD, ZDA are equilateral. Prove that $\angle W P Z + \angle Y P X = 180^\circ$.

- 6** Suppose that x is a real number greater than 1 such that $x^k \lfloor x^k \rfloor$ is an integer for $k = 1, 2, \dots, 2025$. Must x be an integer?

Note: $\lfloor x \rfloor$ denotes the floor function of x , the largest integer less than or equal to x .

BAHASA MELAYU

- 1** Cari semua jangjang aritmetik integer positif $a_1, a_2, \dots$ yang bagi setiap n ,

$$\gcd(a_n, a_{2n}) = n$$

Nota: $\gcd(x, y)$ ditakrifkan sebagai faktor sepunya terbesar bagi x dan y .

- 2** Andaikan ABC sebuah segi tiga tirus dengan $AB < AC$. Andaikan D sebarang titik pada pembahagi dua sama serenjang bagi BC . Bulatan lilit bagi BCD menyalangi AB di E dan AC di F . Garis DE dan DF masing-masing menyalangi garis yang selari dengan BC melalui A di G dan H .

Buktikan bahawa $GHFE$ ialah sisiempat kitaran.

- 3** Terdapat 100 kotak. Di dalam setiap kotak, Ivan meletakkan satu bola merah, kemudian bagi setiap $1 \leq i \leq 100$, dia meletakkan i bola biru ke dalam kotak ke- i . Sekarang, daripada setiap kotak, dia memilih satu bola secara rawak.

Apakah kebarangkalian bahawa dia mendapat bilangan bola merah yang ganjil?

- 4** Andaikan sebuah grid bersaiz $n \times n$, beberapa petak mengandungi bawang putih. Seorang Dracula ingin bergerak dari petak kiri bawah ke petak kanan atas grid tersebut. Dalam setiap langkah, Dracula hanya boleh bergerak ke petak yang bersebelahan, dan dia tidak boleh berada pada petak yang mengandungi bawang putih mahupun bersebelahan dengannya.

Diberi bahawa Dracula berjaya melakukannya, berapakah bilangan maksimum petak yang boleh mengandungi bawang putih?

- 5** Andaikan $ABCD$ sisiempat kitaran dengan pepenjuru AC dan BD bersilang pada P . Titik-titik W, X, Y, Z dibina di luar sisiempat $ABCD$ sehinggakan segi tiga WAB, XBC, YCD , dan ZDA masing-masing ialah segi tiga sama sisi.

Buktikan bahawa $\angle WPZ + \angle YPX = 180^\circ$.

- 6** Andaikan x nombor nyata lebih besar daripada 1 yang $x^k \lfloor x^k \rfloor$ ialah integer bagi setiap $k = 1, 2, \dots, 2025$. Adakah x semestinya integer?

Nota: $\lfloor x \rfloor$ menandakan fungsi lantai, iaitu integer terbesar yang kurang atau sama dengan x .

END OF QUESTION PAPER — KERTAS SOALAN TAMAT
