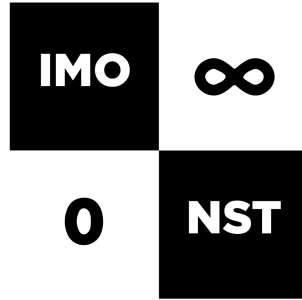




IMO National Selection Test
Ujian Pemilihan Kebangsaan IMO

IMO NATIONAL SELECTION TEST 2 2025

SENIOR CATEGORY

8 NOVEMBER 2025

INSTRUCTIONS:

1. This paper consists of 6 essay questions, each worth 7 points. Answer **ALL** questions.
 2. Each question comes in English Language followed by *Bahasa Melayu*.
 3. The duration of this paper is 4 hours.
 4. Write your answers in the space provided. Show your working to obtain points. You are not required to rewrite the questions.
 5. Only basic utensils and geometric sets are allowed during the exam. Calculators, electronic/internet devices, books, and printed materials are **NOT ALLOWED**.
 6. Discussion, cheating, and guidance are **NOT ALLOWED** during the exam. Any evidence will result in disqualification.
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ENGLISH

1 An elephant is at number 0 and can make moves by going forward k steps, or going backwards 1 step. Determine the smallest m where there is a choice of k such that for all $a \leq 2025$, the elephant can reach number a in $\leq m$ moves.

2 Let ABC be a triangle with circumcenter O and orthocenter H , and let AO intersect the circumcircle of BHC at two points X and Y . Prove that if $\angle AOH = 90^\circ$, then the midpoint of XY lies on the circumcircle of ABC .

3 Let $n \geq 2$ be a positive integer. Determine in terms of n , the maximum value of

$$x_1(x_2 - x_1) + x_2(x_3 - x_2) + \cdots + x_{n-1}(x_n - x_{n-1})$$

over all positive reals $x_1, x_2, \dots, x_n$ satisfying $x_1 \leq x_2 \leq \cdots \leq x_n \leq n$.

4 A *tile* is a collection of orthogonally connected unit squares. Do there exist a set S of 5 pairwise distinct tiles such that:

- All tiles in S are made up by the same number of squares;
- None of the tiles in S are rectangles;
- One can tile an $m \times n$ board using only tiles in S (rotation allowed, reflection not allowed, not using certain tiles is allowed) if and only if 2025 divides both m, n ?

5 Let ABC be an acute triangle with circumcenter O . Let M, N be the midpoints of BC and AO , and the tangent to the circumcircle of ABC at A intersects line BC at D .

Suppose $DM = DN$, find the value of $\angle A$.

6 Let S be a set of 2025 distinct integers. A subset $T \subseteq S$ is called *progression-free* if it contains no three elements in arithmetic progression.

Prove that S always contains a progression-free subset of size at least:

- i. 37.
- ii. 119.

BAHASA MELAYU

- 1** Seekor gajah berada pada nombor 0 dan boleh bergerak dengan maju k langkah, atau mundur 1 langkah. Tentukan nilai terkecil m di mana terdapat suatu pilihan k yang bagi semua $a \leq 2025$, gajah tersebut boleh sampai ke nombor a dalam $\leq m$ gerakan.
- 2** Andaikan ABC sebuah segi tiga dengan pusat-lilit O dan ortopusat H , dan andaikan AO menyalangi bulatan lilit bagi BHC pada dua titik X dan Y . Buktikan bahawa jika $\angle AOH = 90^\circ$, maka titik tengah bagi XY terletak pada bulatan lilit bagi ABC .
- 3** Andaikan $n \geq 2$ integer positif. Tentukan, dalam bentuk n , nilai maksimum bagi

$$x_1(x_2 - x_1) + x_2(x_3 - x_2) + \cdots + x_{n-1}(x_n - x_{n-1})$$

bagi semua nombor nyata positif $x_1, x_2, \dots, x_n$ yang memuaskan $x_1 \leq x_2 \leq \cdots \leq x_n \leq n$.

- 4** Suatu *jubin* ialah sekumpulan petak berukuran unit yang bersambung pada sisi-sisi mereka. Adakah wujud suatu set S yang terdiri daripada 5 jubin berbeza yang:
- Semua jubin dalam S diperbuat daripada jumlah petak yang sama;
 - Tiada jubin dalam S berbentuk segiempat tepat;
 - Seseorang boleh membentuk papan bersaiz $m \times n$ menggunakan hanya jubin dalam S (putaran dibenarkan, pantulan tidak dibenarkan, tidak semestinya menggunakan semua jubin) jika dan hanya jika 2025 membahagi kedua-dua m dan n ?

- 5** Andaikan ABC sebuah segi tiga tirus dengan pusat lilit O . Andaikan M dan N titik-titik tengah bagi BC dan AO , dan garis tangen pada bulatan lilit ABC di A menyalangi garis BC pada D .

Jika $DM = DN$, cari nilai $\angle A$.

- 6** Andaikan S suatu set yang mengandungi 2025 integer berbeza. Suatu subset $T \subseteq S$ dianggap *bebas-janjang* jika ia tidak mengandungi tiga elemen yang membentuk janjang aritmetik.

Buktikan bahawa S sentiasa mengandungi subset bebas-janjang bersaiz sekurang-kurangnya:

- i. 37.
- ii. 119.

END OF QUESTION PAPER — KERTAS SOALAN TAMAT
