

IMO NATIONAL SELECTION TEST 2

SAMPLE PAPER

BEGINNER CATEGORY

Six questions · 7 marks per question · 42 marks

INSTRUCTIONS

1. This paper contains six essay questions. Answer all questions. Each question is worth 7 marks.
 2. Suggested duration: 4 hours.
 3. Show your working clearly. Marks may be awarded for correct reasoning even if the final answer is incorrect.
 4. The English questions are presented first, followed by the Bahasa Melayu questions. The solutions and suggested mark allocations follow both language sections.
 5. No calculator is required.
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ENGLISH

Each question is worth 7 marks.

Question 1

Using the digits 1, 2, 3, 4, form three-digit numbers without repeating a digit. The aim is to find how many of the numbers formed are divisible by 3.

- (a) What must be true about the sum of a number's digits if it is divisible by 3? *[1 mark]*
- (b) Which sets of three digits from 1, 2, 3, 4 have a sum divisible by 3? *[3 marks]*
- (c) How many different three-digit numbers can be made from each qualifying set? Give the total number. *[3 marks]*

Question 2

A child climbs a staircase with 7 steps. On each move, the child may climb either 1 step or 2 steps. Determine the number of different ways the child can reach the top.

- (a) How many ways are there to reach step 1? How many ways are there to reach step 2? *[2 marks]*
- (b) Find the number of ways to reach step 3. Then explain how to determine the number of ways to reach step k , using the numbers of ways to reach steps $k - 1$ and $k - 2$. *[4 marks]*
- (c) Use the rule from (b) to find the number of ways to reach step 7. *[1 mark]*

Question 3

A 6×6 board is coloured like a chessboard, with neighbouring squares having different colours. Two squares at opposite corners are removed. Can the remaining board be covered exactly by 17 dominoes, where each domino covers two neighbouring squares?

- (a) How many squares of each colour does the original board have? *[2 marks]*
- (b) What are the colours of the two opposite corner squares? How many squares of each colour remain after they are removed? *[2 marks]*
- (c) Explain how many squares of each colour every domino must cover, and use this to answer the question. *[3 marks]*

Question 4

A two-digit number has digits whose sum is 11. The tens digit is 3 greater than the units digit. What is the number? Show how you determine both digits.

Question 5

Triangle ABC is isosceles, with $AB = AC$ and $\angle BAC = 40^\circ$. Point D lies on BC , and $\angle BAD = 30^\circ$. Find $\angle ADC$.

Question 6

Prove that the product of any three consecutive integers is divisible by 6.

BAHASA MELAYU

Setiap soalan bernilai 7 markah.

Soalan 1

Dengan menggunakan digit 1, 2, 3, 4, bina nombor tiga digit tanpa mengulang mana-mana digit. Tujuannya adalah untuk mencari bilangan nombor yang terbentuk yang boleh dibahagi dengan 3.

- (a) Apakah syarat bagi jumlah digit sesuatu nombor supaya nombor itu boleh dibahagi dengan 3? *[1 markah]*
- (b) Set digit manakah yang boleh dipilih daripada 1, 2, 3, 4 supaya jumlahnya boleh dibahagi dengan 3? *[3 markah]*
- (c) Berapakah bilangan nombor tiga digit yang boleh dibentuk daripada setiap set yang memenuhi syarat itu? Nyatakan jumlah keseluruhannya. *[3 markah]*

Soalan 2

Seorang kanak-kanak menaiki tangga yang mempunyai 7 anak tangga. Pada setiap langkah, kanak-kanak itu boleh menaiki sama ada 1 atau 2 anak tangga. Tentukan bilangan cara yang berlainan untuk sampai ke puncak.

- (a) Berapakah bilangan cara untuk sampai ke anak tangga 1? Bagaimana pula ke anak tangga 2? *[2 markah]*
- (b) Cari bilangan cara untuk sampai ke anak tangga 3. Kemudian, terangkan cara menentukan bilangan cara untuk sampai ke anak tangga k dengan menggunakan bilangan cara untuk sampai ke anak tangga $k - 1$ dan $k - 2$. *[4 markah]*
- (c) Gunakan peraturan dalam (b) untuk mencari bilangan cara sampai ke anak tangga 7. *[1 markah]*

Soalan 3

Sebuah papan berukuran 6×6 diwarnakan seperti papan catur, dengan petak-petak bersebelahan mempunyai warna yang berbeza. Dua petak pada bucu yang bertentangan dibuang. Bolehkah baki papan ditutupi sepenuhnya oleh 17 domino, dengan setiap domino menutupi dua petak yang bersebelahan?

- (a) Berapakah bilangan petak bagi setiap warna pada papan asal? *[2 markah]*
- (b) Apakah warna bagi dua petak pada bucu yang bertentangan itu? Berapakah bilangan petak bagi setiap warna yang tinggal selepas kedua-duanya dibuang? *[2 markah]*
- (c) Terangkan bilangan petak bagi setiap warna yang ditutupi oleh satu domino, kemudian gunakan maklumat itu untuk menjawab soalan. *[3 markah]*

Soalan 4

Suatu nombor dua digit mempunyai jumlah digit 11. Digit puluhnya adalah 3 lebih besar daripada digit sa. Apakah nombor itu? Tunjukkan cara anda menentukan kedua-dua digit tersebut.

Soalan 5

Segi tiga ABC ialah segi tiga sama kaki, dengan $AB = AC$ dan $\angle BAC = 40^\circ$. Titik D terletak pada BC , dan $\angle BAD = 30^\circ$. Cari $\angle ADC$.

Soalan 6

Buktikan bahawa hasil darab bagi mana-mana tiga integer berturutan boleh dibahagi dengan 6.

SOLUTIONS AND SUGGESTED MARK ALLOCATIONS

Each question is worth 7 marks. Accept any mathematically correct solution with clear reasoning.

Question 1

A number is divisible by 3 if and only if its digit sum is divisible by 3. The four possible sets of three digits are $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, and $\{2, 3, 4\}$; their sums are 6, 7, 8, and 9. Thus $\{1, 2, 3\}$ and $\{2, 3, 4\}$ qualify. Each set has $3! = 6$ arrangements. The total is $6 + 6 = 12$. **Suggested mark allocation:**

- 1 mark for the digit-sum rule.
- 2 marks for listing/checking the four possible sets and their sums.
- 1 mark for identifying the two qualifying sets.
- 2 marks for counting 6 arrangements per set.
- 1 mark for the total, 12.

Question 2

There is 1 way to reach step 1 and 2 ways to reach step 2. For any step k , the last move must come from step $k-1$ by a 1-step move or from step $k-2$ by a 2-step move. Hence the number of ways satisfies $W(k) = W(k-1) + W(k-2)$. The counts for steps 1 through 7 are 1, 2, 3, 5, 8, 13, 21. Therefore, there are 21 ways. **Suggested mark allocation:**

- 1 mark for the count at step 1.
- 1 mark for the count at step 2.
- 2 marks for finding the count at step 3 and showing the calculation.
- 2 marks for explaining the last-move cases and stating the recurrence.
- 1 mark for applying the recurrence to obtain the answer, 21.

Question 3

The 36 squares of the original board are split equally between the two colours, so there are 18 of each. Opposite corners of a chessboard have the same colour. Removing them leaves 16 squares of one colour and 18 of the other. Every domino covers two neighbouring squares, which have different colours, so each domino covers one square of each colour. Seventeen dominoes would cover 17 squares of each colour. This cannot cover a board with 16 squares of one colour and 18 of the other. Therefore, it is impossible. **Suggested mark allocation:**

- 2 marks for 18 squares of each colour.
- 2 marks for identifying the same-coloured corners and the remaining counts 16 and 18.
- 1 mark for stating that each domino covers one square of each colour.
- 2 marks for the contradiction and conclusion.

Question 4

Let u be the units digit. Then the tens digit is $u + 3$. Their sum is 11, so $u + (u + 3) = 11$. Thus $2u = 8$ and $u = 4$. The tens digit is 7, so the number is 74. **Suggested mark allocation:**

- 2 marks for defining the digits and translating the conditions.
- 2 marks for forming the equation.
- 2 marks for finding both digits.
- 1 mark for the answer, 74.

Question 5

Since $AB = AC$, the base angles are equal: $\angle ABC = \angle ACB = (180^\circ - 40^\circ)/2 = 70^\circ$. Since D lies on BC , $\angle ABD = 70^\circ$. In triangle ABD , $\angle ADB = 180^\circ - 30^\circ - 70^\circ = 80^\circ$. Angles $\angle ADB$ and $\angle ADC$ form a straight line, so $\angle ADC = 180^\circ - 80^\circ = 100^\circ$. **Suggested mark allocation:**

- 2 marks for finding the base angles are 70° .
- 1 mark for $\angle ABD = 70^\circ$.
- 2 marks for finding $\angle ADB = 80^\circ$.
- 2 marks for the supplementary-angle step and answer, 100° .

Question 6

Let the three consecutive integers be n , $n + 1$, and $n + 2$. Among any three consecutive integers, one is even and one is divisible by 3. Their product is therefore divisible by both 2 and 3. Since 2 and 3 are relatively prime, the product is divisible by 6. **Suggested mark allocation:**

- 2 marks for factoring into three consecutive integers.
- 2 marks for explaining why one factor is even.
- 2 marks for explaining why one factor is divisible by 3.
- 1 mark for concluding divisibility by 6.