

IMO NATIONAL SELECTION TEST 2

SAMPLE PAPER

PIONEER CATEGORY

Six questions · 7 marks per question · 42 marks

INSTRUCTIONS

1. This paper contains six essay questions. Answer all questions. Each question is worth 7 marks.
 2. Suggested duration: 4 hours.
 3. Show your working clearly. Marks may be awarded for correct reasoning even if the final answer is incorrect.
 4. The English questions are presented first, followed by the Bahasa Melayu questions. The solutions and suggested mark allocations follow both language sections.
 5. No calculator is required.
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ENGLISH

Each question is worth 7 marks.

Question 1

Triangle ABC is isosceles, with $AB = AC = 13$ cm and $BC = 10$ cm. Point D is the midpoint of BC .

- (a) Find BD . [2 marks]
- (b) Explain why AD is perpendicular to BC . [2 marks]
- (c) Use a right triangle to find AD . [3 marks]

Question 2

A set contains the integers 1 through 8. How many ways are there to choose three different numbers whose sum is even?

- (a) How many even numbers and how many odd numbers are in the set? [2 marks]
- (b) What combinations of even and odd numbers can give an even sum when three numbers are added? [2 marks]
- (c) Count the choices in each case and find the total. [3 marks]

Question 3

Five different books consist of two mathematics books and three story books. In how many ways can the books be arranged in a row if the two mathematics books are not next to each other?

- (a) How many arrangements are there without any restriction? [2 marks]
- (b) Count the arrangements in which the two mathematics books are together. [2 marks]
- (c) Subtract to find the number in which they are not together. [3 marks]

Question 4

A square grid is made of 4×4 unit squares. How many squares of all possible sizes are there in the grid?

Question 5

Find all pairs of positive integers (x, y) such that $xy + x + y = 11$.

Question 6

Positive integers x and y satisfy $x + y = 30$. Find the greatest possible value of xy , and state when it occurs.

BAHASA MELAYU

Setiap soalan bernilai 7 markah.

Soalan 1

Segi tiga ABC ialah segi tiga sama kaki, dengan $AB = AC = 13$ cm dan $BC = 10$ cm. Titik D ialah titik tengah BC .

- (a) Cari BD . [2 markah]
- (b) Terangkan mengapa AD berserenjang dengan BC . [2 markah]
- (c) Gunakan segi tiga bersudut tegak untuk mencari AD . [3 markah]

Soalan 2

Suatu set mengandungi integer 1 hingga 8. Berapakah bilangan cara memilih tiga nombor yang berlainan supaya jumlahnya genap?

- (a) Berapakah bilangan nombor genap dan nombor ganjil dalam set itu? [2 markah]
- (b) Apakah gabungan bilangan nombor genap dan ganjil yang boleh menghasilkan jumlah genap apabila tiga nombor dijumlahkan? [2 markah]
- (c) Hitung pilihan bagi setiap kes dan cari jumlah keseluruhannya. [3 markah]

Soalan 3

Lima buah buku yang berlainan terdiri daripada dua buku matematik dan tiga buku cerita. Dalam berapa cara buku-buku itu boleh disusun sebaris jika kedua-dua buku matematik itu tidak bersebelahan?

- (a) Berapakah bilangan susunan tanpa sebarang syarat? [2 markah]
- (b) Hitung bilangan susunan apabila kedua-dua buku matematik itu bersebelahan. [2 markah]
- (c) Tolak untuk mencari bilangan susunan apabila kedua-duanya tidak bersebelahan. [3 markah]

Soalan 4

Suatu grid segi empat sama terdiri daripada petak unit berukuran 4×4 . Berapakah bilangan semua segi empat sama yang boleh dibentuk dalam grid itu?

Soalan 5

Cari semua pasangan integer positif (x, y) supaya $xy + x + y = 11$.

Soalan 6

Integer positif x dan y memenuhi $x + y = 30$. Cari nilai maksimum yang mungkin bagi xy , dan nyatakan bila nilai itu berlaku.

SOLUTIONS AND SUGGESTED MARK ALLOCATIONS

Each question is worth 7 marks. Accept any mathematically correct solution with clear reasoning.

Question 1

Since $AB = AC$, the perpendicular bisector of BC passes through A . As D is the midpoint of BC , $AD \perp BC$. Thus $BD = 5$ and, by Pythagoras in ABD , $AD^2 = 13^2 - 5^2 = 144$. Hence $AD = \boxed{12 \text{ cm}}$. **Suggested mark allocation:**

- 2 marks for $BD = 5$.
- 2 marks for explaining $AD \perp BC$.
- 2 marks for applying Pythagoras correctly.
- 1 mark for $AD = 12 \text{ cm}$.

Question 2

There are 4 even and 4 odd numbers. A sum of three numbers is even if either all three are even, or one is even and two are odd. These cases give $\binom{4}{3} = 4$ and $\binom{4}{1}\binom{4}{2} = 4 \cdot 6 = 24$ choices. The total is $4 + 24 = \boxed{28}$. **Suggested mark allocation:**

- 2 marks for identifying 4 even and 4 odd numbers.
- 2 marks for the two parity cases.
- 2 marks for counting the two cases.
- 1 mark for the total, 28.

Question 3

Without restrictions, the five distinct books can be arranged in $5! = 120$ ways. If the two mathematics books are together, treat them as one block. The block and three story books can be arranged in $4!$ ways, and the two mathematics books can switch places, giving $2 \cdot 4! = 48$. Therefore the number with the mathematics books separated is $120 - 48 = \boxed{72}$. **Suggested mark allocation:**

- 2 marks for $5! = 120$.
- 2 marks for the block count $2 \cdot 4! = 48$.
- 2 marks for subtracting the restricted arrangements.
- 1 mark for 72.

Question 4

The numbers of 1×1 , 2×2 , 3×3 , and 4×4 squares are respectively 16, 9, 4, and 1. Thus the total is $16 + 9 + 4 + 1 = \boxed{30}$. **Suggested mark allocation:**

- 1 mark for each correct count of a square size (4 marks).
- 2 marks for summing the counts.
- 1 mark for the answer, 30.

Question 5

Adding 1 to the equation gives $(x+1)(y+1) = 12$. Since x, y are positive, both factors are at least 2. The factor pairs are (2, 6) and (3, 4), together with their reversals. Therefore $(x, y) = (1, 5), (5, 1), (2, 3), (3, 2)$. **Suggested mark allocation:**

- 2 marks for rewriting as $(x+1)(y+1) = 12$.
- 2 marks for listing the factor pairs.
- 2 marks for converting the pairs to positive x, y .
- 1 mark for the complete answer.

Question 6

Since $(x-y)^2 \geq 0$, we have $(x+y)^2 \geq 4xy$. Hence $900 \geq 4xy$, so $xy \leq 225$. Equality holds when $x = y = 15$, which satisfies the condition. The greatest value is 225. **Suggested mark allocation:**

- 2 marks for using $(x-y)^2 \geq 0$.
- 2 marks for deriving $4xy \leq (x+y)^2 = 900$.
- 2 marks for identifying equality at $x = y = 15$.
- 1 mark for 225.